The mission of the Chardon Local Schools is high achievement for all students, where learning is our most important work.

Course of Study — MATH

Revised March 2022

ADVANCED QUANTITATIVE REASONING

COS — MATH — *Revised March 2022*

Advanced Quantitative Reasoning

Domain: Number and Quantity Standards

Cluster: QUANTITIES

Reason quantitatively and use units to solve problems.

Learning Standards:
N.Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. ★

N.Q.2 Define appropriate quantities for the purpose of descriptive modeling.★ N.Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.★

(Note: Standards N.Q.4-7 are included in this course to be considered for next standards revision.)

Reason quantitatively about numerical data including covariation to solve real-world problems.

Learning Standards:
N.Q.4 Reason, model, and communicate about numerical data building upon previous knowledge of fractions, decimals, percents, scientific notation and estimation and using that knowledge flexibly in a variety of circumstances and a range of number values. ★

N.Q.5 Reason, model, and communicate about proportions including the following: ★ a. Distinguishing between proportional and non-proportional situations. b. Using dimensional analysis. c. Analyzing and comparing growth and decay using absolute and relative change.

N.Q.6 Use models to solve and communicate about contextual financial questions such as budgets, credit card debt, installment savings, amortization schedules, mortgage and other loan scenarios.★

N.Q.7 Identify and explain personal and societal

How Taught?
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- Direct Instruction
- Cooperative Groups
- Stations
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consequences of financial decisions and other scenarios. ★	
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	How Re-Taught? **Re-teaching activities may include, but are not limited to:** ● breaking down concept into smaller components ● presenting the information again in a different way ● Universal Design for Learning principles offering students opportunities to experience and engage material in new and different way ● practice activities such as computer tutorials, games, hands-on activities ● review sessions

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Advanced Quantitative Reasoning

Domain: Algebra Standards

Cluster: SEEING STRUCTURE IN EXPRESSIONS

Interpret the structure of expressions.

Learning Standards:
A.SSE.1. Interpret expressions that represent a quantity in terms of its context. ★
a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Write expressions in equivalent forms to solve problems.

Learning Standards:
A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ (+)

A.SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.★

Cluster: CREATING EQUATIONS

Create equations that describe numbers or relationships.

Learning Standards:
A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations and inequalities arising from linear, quadratic, simple rational, and exponential functions. ★
c. Extend to include more complicated function situations with the option to solve with technology. (A2, M3)

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and

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scales.★
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A.CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.★ (A1, M1)
a. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)

A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.★
d. While functions will often be linear, exponential, or quadratic, the types of problems should draw from more complicated situations. (A2, M3)

Cluster: REASONING WITH EQUATIONS AND INEQUALITIES

Understand solving equations as a process of reasoning and explain the reasoning.

Learning Standards:
A.REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Solve equations and inequalities in one variable.

Learning Standards:
A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Solve systems of equations.

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Learning Standards:
A.REI.6 Solve systems of linear equations algebraically and graphically.

Represent and solve equations and inequalities graphically.

Learning Standards:
A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
A.REI.11 Explain why the x-coordinates of the points where the graphs of the equation y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, making tables of values, or finding successive approximations.
A.REI.12 Graph the solutions to a linear inequality in two variables as a half -plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half -planes.

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Advanced Quantitative Reasoning

Domain: Functions Standards

Cluster: INTERPRETING FUNCTIONS

Understand the concept of a function, and use function notation.

Learning Standards:
F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context.

Learning Standards:
F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include the following: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★ (A2, M3)
F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★
c. Emphasize the selection of a type of function for a model based on behavior of data and context. (A2, M3)
F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★ (A2, M3)

Analyze functions using different representations

Learning Standards:
F.IF.7 Graph functions expressed symbolically and indicate key features of the graph, by hand in simple cases and using technology for more complicated cases. Include applications and how key features relate to characteristics of a situation, making

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selection of a particular type of function model appropriate.★
F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (A2, M3)

Cluster: BUILDING FUNCTIONS

Build a function that models a relationship between two quantities.

Learning Standards:
F.BF.1 Write a function that describes a relationship between two quantities.★
a. Determine an explicit expression, a recursive process, or steps for calculation from context.
b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. (A2, M3) (+)
c. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Cluster: LINEAR, QUADRATIC, AND EXPONENTIAL MODELS

Construct and compare linear, quadratic, and exponential models, and solve problems.

Learning Standards:
F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential

functions.★ **b**. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. **c.** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **F.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★ (+) **F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. ★ (A1, M2) **Interpret expressions for functions in terms of the situation they model.** **Learning Standards:** **F.LE.5** Interpret the parameters in a linear or exponential function in terms of a context.★ **Cluster: TRIGONOMETRIC FUNCTIONS** **Extend the domain of trigonometric functions using the unit circle.** **Learning Standards:** **F.TF.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **Model periodic phenomena with trigonometric functions.** **Learning Standards:** **F.TF.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★	
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Domain: Geometry Standards

Cluster: CONGRUENCE

Experiment with transformations in the plane.

Learning Standards:

G.CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not, e.g., translation versus horizontal stretch.

G.CO.3 Identify the symmetries of a figure, which are the rotations and reflections that carry it onto itself.

a. Identify figures that have line symmetry; draw and use lines of symmetry to analyze properties of shapes.

b. Identify figures that have rotational symmetry; determine the angle of rotation, and use rotational symmetry to analyze properties of shapes.

G.CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using items such as graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Cluster: SIMILARITY, RIGHT TRIANGLES, AND TRIGONOMETRY

Prove and apply theorems both formally and informally involving similarity using a variety of methods.

Learning Standards:

G.SRT.4 Prove and apply theorems about triangles. Theorems include but are not restricted to the following: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

G.SRT.5 Use congruence and similarity criteria for triangles to solve problems and to justify relationships in geometric figures that can be decomposed into triangles.

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Define trigonometric ratios, and solve problems involving right triangles.

Learning Standards:
G.SRT.8 Solve problems involving right triangles.★

Cluster: CIRCLES

Find arc lengths and areas of sectors of circles.

Learning Standards:
G.C.5 Find arc lengths and areas of sectors of circles.

Cluster: GEOMETRIC MEASUREMENT AND DIMENSION

Understand the relationships between lengths, areas, and volumes.

Learning Standards:
G.GMD.6 When figures are similar, understand and apply the fact that when a figure is scaled by a factor of k, the effect on lengths, areas, and volumes is that they are multiplied by k, k^2 , and k^3 , respectively.

Cluster: MODELING WITH GEOMETRY

Apply geometric concepts in modeling situations.

Learning Standards:
G.MG.1 Use geometric shapes, their measures, and their properties to describe objects, e.g., modeling a tree trunk or a human torso as a cylinder.★
G.MG.2 Apply concepts of density based on area and volume in modeling situations, e.g., persons per square mile, BTUs per cubic foot.★
G.MG.3 Apply geometric methods to solve design problems, e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios.★

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Domain: Statistics and Probability Standards

Cluster: INTERPRETING CATEGORICAL AND QUANTITATIVE DATA S.ID

Summarize, represent, and interpret dataG on a single count or measurement variable.

Learning Standards:

S.ID.1 Represent data with plots on the real number line (dot plotsG, histograms, and box plotsG) in the context of real-world applications using the GAISE model.★

S.ID.2 In the context of real-world applications by using the GAISE model, use statistics appropriate to the shape of the data distribution to compare center (median and mean) and spread (mean absolute deviationG, interquartile rangeG, and standard deviation) of two or more different data sets. ★

S.ID.3 In the context of real-world applications by using the GAISE model, interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). ★

S.ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.★

Summarize, represent, and interpret data on two categorical and quantitative variables.

Learning Standards:

S.ID.6 Represent data on two quantitative variables on a scatter plotG, and describe how the variables are related.★

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. (A2, M3)

b. Informally assess the fit of a function by discussing residuals. (A2, M3)

c. Fit a linear function for a scatterplot that suggests

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a linear association. (A1, M1)

Interpret linear models.

Learning Standards:
S.ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★
S.ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.★
S.ID.9 Distinguish between correlation and causation.★

Cluster: MAKING INFERENCES AND JUSTIFYING CONCLUSIONS

Understand and evaluate random processes underlying statistical experiments.

Learning Standards:
S.IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.★
S.IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?★

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Learning Standards:
S.IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★
S.IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.★
S.IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if

differences between sample statistics are statistically significant.★
S.IC.6 Evaluate reports based on data.★

Cluster: CONDITIONAL PROBABILITY AND THE RULES OF PROBABILITY

Understand independence and conditional probability, and use them to interpret data.

Learning Standards:
S.CP.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★
S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.★
S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.★

Use the rules of probability to compute probabilities of compound events in a uniform probability model

Learning Standards:
S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.★
S.CP.7 Apply the Addition Rule, P(A or B) = P(A) +

P(B) − P(A and B), and interpret the answer in terms of the model.★

Cluster: USING PROBABILITY TO MAKE DECISIONS

Calculate expected values, and use them to solve problems.

Learning Standards:
S.MD.1 Define a random variableG for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distributionG using the same graphical displays as for data distributions.★
S.MD.2 Calculate the expected valueG of a random variable; interpret it as the mean of the probability distribution.★

Calculate expected values, and use them to solve problems.

Learning Standards:
S.MD.3 Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.★
S.MD.4 Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?★

Use probability to evaluate outcomes of decisions.

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Learning Standards:

S.MD.5 Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.★ a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant. b. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.

S.MD.6 Use probabilities to make fair decisions, e.g., drawing by lots, using a random number generator.★

S.MD.7 Analyze decisions and strategies using probability concepts, e.g., product testing, medical testing, pulling a hockey goalie at the end of a game.★

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